Living in harmony: nominalism and the explanationist argument for realism

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Abstract

According to the indispensability argument, scientific realists ought to believe in the existence of mathematical entities due to their indispensable role in theorising. Arguably the crucial sense of indispensability can be understood in terms of the contribution that mathematics sometimes makes to the super-empirical virtues of a theory. Moreover, the way in which the *scientific realist* values such virtues in general, and draws on *explanatory virtues* in particular, ought to make the realist ontologically committed to abstracta. This paper shows that this version of the indispensability argument glosses over crucial detail about how the scientific realist attempts to generate justificatory commitment to unobservables. The kind of role that the platonist attributes to mathematics in scientific reasoning is compatible with nominalism, as far as scientific realist arguments are concerned.

1 Introduction

The contemporary proponents of the indispensability argument for mathematical platonism argue that putting together scientific realism and mathematical nominalism results in "a marriage made in hell". (Colyvan, 2006) This conjugal tension originates from the *fundamental unity* that the scientific realist and the platonist arguments purportedly exhibit. This can be seen by comparing (i) the realist's explanationist argument for her commitment to the theoretically indispensable unobservable posits of science, with (ii) the platonist's indispensability argument for her commitment to the indispensable mathematical posits. The contemporary proponents of platonism argue that sympathising only with (i) borders on inconsistency, for the argument (ii) is allegedly powered by essentially similar admiration for the way the theoretical posits of science—suitably broadly construed, to include applied mathematics—can enhance a theory's virtue. Scientific realism and mathematical nominalism do not make a happy, stable couple: the rejection of the platonist's indispensability argument is based on *mere prejudice* against the abstracta, and that prejudice could be extended to equally undermine the realist's commitment to unobservables.

This paper aims to provide a healthy dose of marriage counselling by opposing this contemporary spin on the indispensability argument.[1] It is argued that on a closer analysis the purported unity of the realist and platonist arguments turns out to be rather superficial, and incapable of giving rise to any serious tension in the marriage of scientific realism and mathematical nominalism. A closer inspection of the two arguments (i) and (ii) reveals that the purported unity is achieved only by glossing over crucial detail about how the scientific realist attempts to generate justificatory commitment to unobservables. By having a closer look at the realist arguments, in particular, we can make distinctions that help us to understand better the potential contribution of mathematics to theoretical, super-empirical and explanatory, virtues. It turns out that the nature of realist arguments actually allow for a natural discriminatory distinction to be drawn between abstract and concrete theoretical posits, with respect to realist commitment, that goes beyond mere prejudice. The kind of role that the platonist attributes to mathematics in scientific reasoning is in the end compatible with nominalism, as far as scientific realist arguments are concerned.

The paper proceeds as follows. The next section (§2) provides an exposition of the Quine-Putnam-Colyvan indispensability argument, drawing attention to its relation to scientific realism. After that, the argumentative foundation of scientific realism itself will be considered, with an emphasis on the precise role that inference to the best explanation plays in general (§3). This analysis will already show that the platonist argument is inadequate as it stands, and that there is ample logical room for specific realist arguments that are compatible with *both* nominalism *and* the premises of the platonist argument. The following section (§4) explores such a realist argument of particular significance, the No-Miracles Argument, in some detail. The final two sections further elaborate and illustrate my position by providing a commentary on the recent debate on the proper understanding of the indispensability of mathematics. I will take issue with the whole of current state of debate which, I claim, has framed the indispensability question in more or less irrelevant terms.

2 Quine-Putnam-Colyvan Indispensability Argument

Colyvan (2001a) is a sustained defence of the indispensability argument for the existence of mathematical objects. Colyvan's eminent role in analysing, sharpening and maintaining the argument originating from the writings of Quine and Putnam legitimises dubbing it *Quine-Putnam-Colyvan* (henceforth QPC) indispensability argument. Colyvan formulates QPC explicitly as follows:

(P1) We ought to have ontological commitment to all (and only) the entities that are indispensable to our best scientific theories.

(P2) Mathematical entities are indispensable to our best scientific theories.

(C) We ought to have ontological commitment to mathematical entities.

The second premise is meant to capture the seemingly uncontroversial fact that science heavily relies on mathematics. It needs to be made much more precise, of course, given the modal dimension of this claim: is the assumption that science cannot dispense with mathematics *categorically*, or that it cannot dispense with mathematics *in practice*? If mathematics is strictly, or logically speaking, dispensable, the claim seems to hang on some judgement about the mathematized theories being *better*, so that mathematics is indispensable only to our *best* science. QPC opts for the latter reading of indispensability: the claim is that dispensing with mathematics (if at all possible) would result in theories which are "less attractive". The argument thus hangs on (i) what we take theoretical attractiveness to consist in, and (ii) how contribution to such attractiveness is linked to ontological commitment.

QPC, as propounded by Colyvan, for example, weds these questions explicitly to arguments for scientific realism. It is maintained that at least part of the role of mathematics in theorising is akin to the role of unobservable concrete theoretical posits in that both can contribute to the super-empirical virtues of a theory.[2] When it comes to choosing between empirically equivalent theories, these theoretical virtues matter. Scientific realists take theoretical virtues particularly seriously, claiming that their philosophical significance goes well beyond pragmatics. Granting this realist attitude towards theoretical virtues now enables the advocates of QPC to generate some prima facie motivation for the first premise. Surely, they say, we ought to have an impartial ontological attitude towards the generators of the super-empirical virtues of a theory, regardless of whether they are concrete or abstract objects.

This connection to scientific realism is at the heart of the argument. Platonism is seen almost as *a corollary* of scientific realism; one cannot (at least not easily) have the latter without the former. Witness Colyvan:

> The [QPC] is the argument for treating mathematical entities on a par with other theoretical entities of our best scientific theories.
>
> ...
>
> [T]he proper way to understand this argument is as putting pressure on the viability of the marriage of scientific realism and mathematical nominalism.
>
> ...
>
> Unless one is careful about how the Quine-Putnam argument is disarmed, one can be forced to either mathematical realism or, alternatively, scientific instrumentalism. (Colyvan, 2006, p. 225)

It is this way of putting the argument in terms of the scientific realist's commitments that I want to focus on here. Whatever else may be right or wrong about QPC, I will argue that it is a mistake to draw such parallel between the platonist argument, vis-à-vis the indispensability of complex numbers, say, and the realist argument, vis-à-vis the indispensability of electrons, for example.

Let us now consider the platonist argument further. How exactly is the connection between realism and platonism being argued for? Much turns on the plausible but rough impression that "the realism-antirealism debate can be characterised in terms of the acceptance or rejection (respectively) of inference to the best explanation" (Colyvan, 2006, p. 226). Colyvan adopts a general characterisation of an indispensability argument from Field (1989), and views inference to the best explanation "as a special case of the indispensability argument" (2001a, p. 7). Although the two argument forms are strictly speaking different, Colyvan suggests that given their affinity, "those who accept inference to the best explanation are at least sympathetic to this style of argument" (2001a, p. 8). Furthermore, given that "mathematical entities surely feature prominently in various explanations", he asserts that

> [QPC] puts pressure on the marriage of scientific realism and nominalism … *because the style of argument is one which scientific realists already endorse*. (Colyvan, 2006, pp. 227–8, my emphasis)

The idea here is that there is a kind of *unity* to the indispensability argument for platonism and the inference-to-the-best-explanation argument for scientific realism—or to the mathematical and physical posits, if you like—that makes a mathematical nominalist / scientific realist combination awkward at the very least, if not outright inconsistent. But what exactly does "the style of argument endorsed by scientific realists" refer to? By taking a closer look at realist arguments it will be argued in the next section that the platonist is painting with too broad a stroke here.

But before moving on, let's take a step back and look at the origins of this line of thought. As is well known, the roots of platonists' unity intuition can be found in Quinean confirmational holism, according to which the unit of confirmation is nothing short of the whole theoretical scheme we possess. If one's realist argument leads one to include electrons into one's ontology, then the same evidence should double as evidence for the existence of mathematical entities purportedly referred to in the electron theory. Another leaf from Quine's book is his naturalism—the principled reverence for the ontological commitments of our best science, not to be criticised or overturned by any 'first philosophy'. Hence, any philosophical qualms about the difference between concrete and abstract entities, say, are quashed in the face of uniform quantification over both kinds in our best theories, or of their uniform indispensability (in the contemporary parlance).

This combination of confirmational holism and naturalism has been criticised on the basis of actual scientific practice, e.g. by Maddy (1992). Maddy's argument is problematic (cf. Colyvan, 2001a), and I will next pursue a different kind of practice-of-science objection, by focusing purely on the connection of scientific realism and platonism. The practice of actual science matters greatly for the realist, and admittedly many realist arguments understand this practice as being fundamentally *abductive* in a way that incorporates the super-empirical virtues that QPC capitalises on. It will be next seen, however, that the way these virtues enter the best contemporary scientific realist arguments does *not* lend support to platonism.

3 What Scientific Realism is, and Why It Matters

The present-day campaigners for QPC don't say much about *what scientific realism is*. One mainly finds broad allusions to (i) the general acceptance of inference to best explanation (IBE) by scientific realists of both nominalist and platonist stripe; (ii) the role of super-empirical virtues in theory choice. In his discussion of theoretical attractiveness, after offering the standard list of theoretical desiderata[3] going beyond mere empirical adequacy and consistency, Colyvan says that "despite the notorious difficulties involved in explicating what we mean by such terms as 'simplicity' and 'elegance,' most scientific realists at least do look for such virtues in our best theories." (2001a, p. 79) And, since applied mathematics can allegedly contribute to these virtues on a par with the concrete theoretical posits of science, the realist is insistently invited to extend her ontological commitments beyond the latter.

I now wish to pursue the following worry about this line of thought: the connection between realism and the theoretical utility of mathematics is cast in too general terms, for we can easily conceive how a realist argument could accommodate mathematical contribution to theoretical virtues without entailing any unificatory pressure at the level of ontological commitments. To spell this objection out we need to look at the role of theoretical virtues and inference to the best explanation in the contemporary realism debate, first in the abstract (this section) and then in a particular realist argument (§4). My conclusion will be that the attempt to motivate the premises of the indispensability argument by an ambiguous reference to scientific realist commitments fails by virtue of grossly simplifying the mode of employment of IBE in the realist arguments.

But first: what *is* scientific realism? This is a huge question, of course, at the centre of massive literature. I want to characterise scientific realism here in general terms, faithfully to the spirit of this multifaceted position. What is the best way to do this? It is commonplace to simply equate scientific realism with the belief that our best (most successful) theories are *approximately true*. This is intuitive but inadequate by itself; we need to be told also what is meant by 'approximately true'. In particular, the platonist could argue (as an anonymous referee reminded me) that the realist is committed to mathematical entities purely by virtue of this characterisation of realism, unless, of course, the notion of approximate truth is delineated so that the truth content of an approximately true theory only concerns concrete theoretical posits. But nothing has been said in this regard by the realist; the issue of platonism is simply one which scientific realists have not been (generally speaking) concerned with.

Rather than characterising realism in terms of approximate truth, it is better to appeal to some typical features of realist arguments: what are the argumentative resources employed, and what is being argued for? Most contemporary realists see themselves as articulating and defending a position that is more or less defined by the anti-realist opposition: the sceptical arguments from underdetermination and radical past theory shifts.[4] As far as the latter is concerned, our best understanding of the history of science forces the realist to aim at some sort of approximate truth, on pain of inconsistency. The *justificatory challenge* of responding to the underdetermination problem, on the other hand, calls for an analysis of the scientific method, and its inductive nature and success. The opposition here is typically construed as something like constructive empiricism, featuring a rather guarded attitude towards inductive reasoning about the unobservable.[5]

It is exactly in connection to the justificatory challenge that realists typically invoke arguments turning on some alleged feature(s) of the actual scientific practice, its success and/or its abductive methodology. For example, a typical realist thesis is that (very broadly speaking) (a) scientific method can be described, to an extent, as being infested with reasoning via inference to the best explanation, and (b) this method, thus understood, can be justified, to an extent, to be truth-conducive to the unobservable claims of science. Inference to the best explanation enters the argument as being central to the method of science, and the super-empirical virtues are essential in fixing the 'best' in 'inference to the best explanation', given some context and background beliefs. Such a model of scientific inference and confirmation has been developed by Lipton (2004), for example, and other realists approve on the whole.

Now, there is an important distinction to be recognised here. Lipton (2004) is particularly clear on this, taking great care to emphasise that there are two separate philosophical problems here, regarding the inductive method of science; there are two 'levels' to the typical realist argument, as construed above. Although the realist is ultimately concerned with the epistemological warrant for the beliefs produced by the abductive method—that is, with *justification* of realism—there is a conceptually independent and prior challenge of *describing* the abductive character of the method and the role of theoretical virtues in particular. There can be considerable agreement among different realists about the descriptive model of IBE, without much accord regarding the preferred treatment of the justificatory challenge. For example, Lipton himself denounces the global explanationism of the so-called 'No Miracles' argument (cf. section 4, below) favoured by the contemporary arch-realist Psillos, whilst nevertheless retaining a realist attitude towards a particular class of abductive inferences.

Coming back to the QPC argument, what is the significance for the issue at hand of thus separating the different components of the typical realist thesis? In so far as the realist's ontological commitments are concerned, it is important to recognise how these ultimately depend on (i) the justificatory argument employed, and (ii) the conclusions of the scientific inductions thus justified.

Recall that a justificatory argument is needed to sanction the ampliative inferences of our best scientific theories as licit: we are warranted in believing the conclusions of these inferences, even when they are about the unobservable. The explanationist argument for realism tries to provide such warrant by establishing the truth-tracking reliability of inference to the best explanation. This, according to the explanationist, is the general method of inference actually employed in making scientific inferences. Once such justificatory argument is in place, the scientific realist is ontologically committed to all those entities that figure in the conclusions of the reliable scientific inferences. We know that these inferences are about electrons, the DNA, et cetera, and unprejudiced ontological commitment ought to take equally seriously complex numbers and other mathematical entities, if the inferences feature such abstracta similarly *in their conclusions*. The question regarding (ii), above, then is: are numbers on a par with (concrete) unobservables, in the conclusions of scientific inferences?

On the other hand, it is not enough that our *description* of the scientific inference-making makes apparent reference to mathematical entities. And here we come to see how the platonist's vague reference to the role of mathematics vis-à-vis super-empirical virtues and IBE is glossing over crucial issues. Our best understanding of the truth-tracking reliability of the scientific method *can* make reference to the role of mathematics in all this, *without* immediately entailing any pressure on the nominalist. Detailing the role of mathematics in the inductive method belongs to the descriptive phase of the realist argument, in the first place, and it can be independent from the justificatory phase. Mathematics can figure indispensably in scientists' determination of super-empirical virtues, but if the conclusions of the justified scientific IBEs are not about the existence of mathematical objects (a premise we shall return to in the next section), the realist need not feel any pressure to extend her ontological commitments. All she needs is a justificatory argument for the reliability of these scientific IBEs that does not itself hinge on the existence of mathematical entities.

The point is that since the scientific inferences that concern the existence and properties of concrete unobservables are inductive, the possibility arises that the reliability of (at least some of) these inferences can be argued for by the realist without granting existence to the mathematical entities that nevertheless may contribute to the inductive method, according to the realist's description of that method. The realist's ontological commitments need only include those entities that scientists themselves posit in their best theories, *by virtue of using* explanatory inferences the reliability of which the realist (somehow) argues for. Exactly *how* mathematics can be thus reliably employed in scientific reasoning may be somewhat of a mystery, of course. If the platonist had an explanation for this dimension of the applicability of mathematics—something akin to the No Miracles explanation, perhaps—then the indispensability of mathematics for the method at the descriptive level could count in favour of platonism (provided that the nominalist could not provide an alternative, equally good explanation). But such an explanation, even if it exists, is not part and parcel of the argument presently criticised. As far as the indispensable role of mathematics is concerned, the applicability of mathematics is left equally unaccounted for by both the nominalist and the platonist (as pointed out by Colyvan (2001b), for example).

This general analysis of the contemporary realist gambit points to some shortcomings of the platonist argument. In particular, one runs a risk of a gross oversimplification of subtle issues by fixating on a kind of argumentative unity of the realist and platonist positions that depends on ignoring the details of the the two positions. Colyvan's example-based indications that mathematics "plays an active role in many of the theories that make use of it", that it "makes important contributions to all of the desiderata of good theories" (2001a, p. 80), simply aren't enough. Colyvan argues that given the role of mathematics as described by him,

> ...the burden of proof is on [the nominalist to show that nominalization] does not result in a reduction of the virtue... (Colyvan, 2001a, p. 80)

The above analysis turns the tables, however, for not enough has been said about the role of mathematics in the justificatory phase of the argument. Nevertheless, if mathematics really is (or turns out to be) a significant, ineliminable element in the description of the inductive method of science, it is possible that the platonist intuitions are fulfilled on closer inspection of actual justificatory arguments espoused by realists. The next section will nullify this hope by considering a particular form of realism together with the important outstanding question: are numbers on a par with unobservables in the conclusions of scientific inferences?

4 Super-empirical Virtues and the No-Miracles Argument

A charitable reading of QPC focuses on full-blown theory realism defended via an IBE-based justificatory argument.[6] In particular, one should focus on the IBE-based 'No Miracles' argument, as defended by Psillos (1999), for example, for it makes the strongest, overarching use of IBE and all the standard super-empirical virtues. Furthermore, this argument does not only defend the reliability of the scientists' inferences, but of inference to the best explanation more generally, as the No Miracles argument is arguably justified by virtue of being itself a philosophical inference of the same form. Perhaps such an overarching sanctioning of IBE brings along the kind of tension that QPC plies us with?

Hardly. There are two (related) points to be made here, one about cumulative 'background dependency' of scientific reasoning, and another about the difference between scientific and mathematical inferences.

A central tenet of Psillos's elaboration of Boyd's explanationist defence of realism is the *background dependence* of scientific inferences.

> ...the best explanation of the instrumental reliability of scientific methodology is that background theories are approximately true. These background scientific theories have themselves been typically arrived at by abductive reasoning. Hence, it is reasonable to believe that abductive reasoning is reliable... (Psillos, 1999, p. 80)

This most sophisticated and elaborate version of the No Miracles argument turns on the cumulative background dependence of scientific IBE inferences upon the conclusions of earlier inferences of that form. It is *this* kind of background dependency that gives rise to the potential 'miracle': if the earlier inferences have not, by and large, resulted in true conclusions—that is, if the inference-making of that kind has not been reliable—then it is arguably very difficult to understand how the heavily background dependent current inferences can lead to successful predictions.

This background dependency is part of the reason why the No Miracles Argument (thus understood) resists the kind of parallel that QPC draws: the role of mathematics in abductive reasoning is *not* on a par with the earlier scientific conclusions about concrete unobservables, as far as the 'miracle' is concerned. We can distinguish between the 'broad' inferential background, part of which is mathematics, and the 'narrow' background that is comprised of the conclusions of earlier scientific inferences regarding the existence and properties of concrete theoretical posits. The 'miracle' of the No Miracles Argument—the predictive and instrumental success of science—is construed more specifically as the inconceivability of reaching the known levels of success by building on a false *narrow* background by using the inferential method (IBE) which led to that background.[7] By contrast, we can nicely explain away the 'miracle' by assuming the reliability of the inferential method itself. It first leads to true conclusions (about electrons, say) at an earlier stage of scientific reasoning, from a set of previously accepted truths, and it later leads to further true conclusions, including some novel predictive successes, about matters (involving quarks, say) building on those earlier truths. This realist argument does not require that mathematics must be latching onto reality in any platonist sense in order to explain the success of science. Mind you, this doesn't mean that the realist is able to explain the success of scientific method on the whole; exactly how we manage to use mathematics in inductive reasoning remains a mystery, but the platonist fares no better in this regard.

The earlier scientific inferences, together with the 'static' auxiliaries (mathematics, logic, basic principles), form the broad background for creating and evaluating competing theories. The indispensability argument, in QPC flavour, contends that since 'mathematical posits' are sometimes on a par with the physical ones—in terms of determining the super-empirical virtues of a theory—we are defending not only the reliability of the method, but also the truth of the *whole* background *including all the auxiliaries*. But this is not required by the realist argument; the only bit of the background that needs to be (approximately) true is the bit about concrete unobservables generated by the earlier inferences of the same form. This is enough for the realist to consistently argue for her realism (leaving aside any worries about the obvious circularity involved in stepping to the IBE meta-level). In effect, one can argue for the reliability of the IBE-infested scientific methodology via Boyd-Psillos argument without sliding into platonism, since none of the narrow background scientific inferences draw the conclusion that mathematical entities exist! This is now the second point that I want make against QPC, regarding the difference between scientific and mathematical inferences. There is a natural intuition behind this: the way electrons become part of the corpus of scientific knowledge and talk is different from the way that complex numbers do. Talk of 'mathematical posits' on a par with 'physical posits' is philosophers' talk, of course, not scientists'! Mathematical method is deductive and cannot be assimilated with the explanation-driven inductive method that arguably rules scientific inferences. No mathematical entity has ever been *introduced* as the best explanation of some (mathematical, or physical) phenomena. No abductive inference has ever brought new mathematical facts to our attention. Mathematical knowledge may play a part in abductive inference-making and evaluation of the explanatory worth of alternative hypotheses, but this belongs only to the *broad* inferential background and is hence outside of the realist's justificatory gambit.

The platonist is quick to point out that scientists often conclude their reasoning along the lines of, for example, "So there exists a differentiable vector-valued function that has as its domain the space-time manifold." The conclusions of scientific inferences typically purportedly refer to mathematical entities, of course. My point is that the mathematical entity here, a function, gets *first introduced* as a part of a mathematical theory (of differential geometry) and not as the best explanation of some phenomena. This piece of mathematics then gets *applied*.

Contrast this to the way that the concept of (physical) curvature of spacetime, for example, is introduced. Speaking of the reliability of inference to the best explanation *simpliciter* is actually somewhat ambiguous here. A naïve reading of inference to the best explanation in the case of general relativity, for example, might suggest that the theory, mathematics and all, is (approximately) true as the best explanation of the phenomena.[8] But this is not how IBE functions in theorising. We need to pay close attention to the complex reasoning that led to this theory (it took Einstein over ten years, of course), and appreciate the various inferences that concern purely the concrete world and are part of that reasoning (e.g. the basic physical principles like the equivalence principle). By drawing the distinction between narrow and broad inferential background I argued above that the realist's appeal to the cumulative background dependency of scientific reasoning does not aim to justify the inference to the truth of an explanatory statement or theory *in toto*, but rather the reliability of IBE vis-à-vis the concrete theoretical posits.[9]

The following two sections will elaborate on the above arguments by considering the status of the recent debate and by looking in closer detail at the kind of utility that platonists accord to mathematics.

5 Super-empirical Virtues and Mathematics

The Quine-Putnam-Colyvan indispensability argument has recently come under fire from nominalists who have drawn antithetical distinctions regarding mathematics' contribution to super-empirical virtues. In this section I review this challenge and platonist responses to it, and flag the ensuing debate as unprofitable. The next section (§6) then elaborates on this verdict by commenting on the platonist suggestion that mathematics can be genuinely explanatory of physical phenomena.

Many have expressed worries about mathematical utility not being *of the right* *kind* for science to warrant the conclusion of QPC. Papineau states that "what guides us in our choice of scientific theories is not computational simplicity but *physical* simplicity" (1993, p. 196, my emphasis), and Melia (2000) makes a similar point in terms of simplicity of the *world*, as opposed to a *theory*:[10]

> I accept that considerations of simplicity play an important role in theory choice. But I prefer the hypothesis that makes *the world* a simpler place. For sure, all else being equal, I prefer the theory with the simpler ontology. For sure, all else being equal, I prefer the theory that postulates the least number of fundamental properties and relations. But the simplicity I value attaches to the kind of world postulated by the theory—not to the *formulation* of the theory itself. (Melia, 2000, p. 473)

The worry here is that mathematics is only an efficient codification, a language that simplifies the presentation of the theory, but not its content about the concrete world that ultimately matters. Postulating quarks, for example,

> ... genuinely makes the *world* a simpler place. Under the quark hypothesis, various objects in the particle zoo do exist *in virtue of* the existence, properties and relations of quarks. (*ibid.*)

But not so for numbers, for although "mathematical objects may simplify our theories, they do not simplify our account of the world." (*ibid.*)[11] Hence, the challenge is set: when assessing the contribution of mathematics to the super-empirical virtues of a theory, it is not enough for the platonist to point out that mathematics can make a theory more attractive; it needs to make a theory more attractive in the right way, namely, the way that concrete theoretical posits simplify, unify and (more generally) increase the explanatory power of our account of the world.

Colyvan (2002) reflects on this challenge and provides a response. He insists that mathematics plays "an active role" in theorising, and is not just a tool of expression or calculation; i.e. it makes bona fide contributions to the desiderata of a good theory in a way that matters for the scientific realist.[12] Effectively Colyvan agrees with Melia that not any kind of increase in attractiveness counts, but he purports to give examples that "by Melia's own light, present good reason to embrace platonism" (p. 69). There is no need for us to give even a cursory glance here at these particular examples concerning the unificatory and explanatory role of complex numbers and the Minkowski metric. We shall return to the issue of whether mathematical objects can be genuinely explanatory (and whether it matters for the realist) in the next section (§6).

The current state of debate is characterised by the following agreements and disagreements. Melia (2002) remains unimpressed by the instances of mathematical virtue presented at him, but he does concede that *if* mathematics was shown to bring in theoretical utility in the same measure as the physical content does, *then* it would count in favour of the platonist. Colyvan and Baker (2005), on the other hand, claim to give examples of exactly the required kind. Melia rebuts Colyvan's (2002) examples by identifying the role of mathematics as (i) merely "picking out", or "indexing" some truly explanatory *physical* assumptions; (ii) merely being useful (or indispensable-in-practice) in "expressing", and "discovering" *physical* commonalities that would otherwise remain hidden or that couldn't be expressed. As far as these examples go, mathematics *per se* neither unifies nor explains. Baker's example awaits a response, but both parties conclude that a single case study in which mathematics can be seen as enhancing the explanatory power of a physical theory would resolve the debate in favour of the platonist.

In my view the focus of this debate is well and truly misplaced. Although I'm highly sympathetic to Melia's rebuttal of the alleged platonist implications of Colyvan's examples, in my view the strength of his conclusions does not lie squarely in the contrast between "the world" and "the theory", which might even be question begging provided that we begin with suitable (Quinean) realist premises (cf. note 12). Rather, we should arrive at these conclusions by a detailed consideration of the contemporary realist arguments as laid out in the previous section. The realist commitments of the explanationist arguments in question arise through justification of the reliability of the scientific exercise of inference to the best explanation. The structure of the realist's justificatory argument is such that the relevant commitment-inducing theoretical posits are those the existence of which is postulated (at some stage or another) *exclusively* in order to give the best explanation of something. And in science-as-we-know-it those are the concrete physical entities and their properties—this provides the required asymmetry between the concrete and the abstract without the risk of question begging.

This change in focus is important wholly irrespective of what one thinks about Melia's response to the particular examples put to him. Its importance lies in framing the issue in a way that is not dependent on such difficult matters as to whether or not mathematics can be explanatory in a sense that influences theory choice. Thus it steers the focus away from the kind of philosophical ping-pong that we are in danger of getting into: searching for and rebutting singular case studies which—when combined with a particular understanding of what explanation *is*—might shift the balance one way or another. Baker (2005) offers one such case study which will be considered next to elaborate on the present verdict.

6 Mathematics and Explanatory Power

Much of the debate has recently been conducted in explicitly explanatory terms, effectively turning on the question of whether mathematics can yield genuine explanatory power (outside mathematics). Colyvan thinks it can do but Melia claims that the real explanatory work in his examples is done by the physical properties that are only mathematically expressed, or "indexed". Hence, for example, the FitzGerald-Lorentz contraction is a *physical-geometrical* explanation in relativity, rather than a mathematical one, although the geometrical facts are indispensably—as far as relativistic theorising is concerned, perhaps—codified in mathematics. Recently Baker (2005) has joined the debate on the platonist side, arguing for a number theoretical explanation of the periodical life-cycle of cicada. I will now briefly comment on this example to elaborate on the previous section and to illustrate my general take on the issues at hand.

Baker (2005) claims that the best available scientific explanations for the idiosyncratic life-cycle lengths of the North-American cicada—exactly 13 or 17 years, depending on the geographical area—incorporate elementary number theory in an indispensable fashion that cannot be swept under the carpet as merely indexing some underlying physical facts. Hence, since a purely mathematical component is arguably "both essential to the overall explanation and genuinely explanatory in its own right," it supports the case for QPC:

> [T]he explanation of the prime cycle lengths of periodical cicadas using number theory is one example of [genuine mathematical explanations of physical phenomena]. If this is right, then applying inference to the best explanation in the cicada example yields the conclusion that numbers exist. (Baker, 2005, p. 236)

Without going into any detail about the explanandum or the suggested explanations, I just note the following about "applying inference to the best explanation" in this case.[13] Baker's example can be used to illustrate nicely the distinction made above between the 'broad' and the 'narrow' background (cf. §4). In his formalisation of the basic structure of the evolutionary explanation, there are (i) laws and constraints concerning the concrete world that result from earlier scientific investigations (narrow background), which are (ii) used together with a number theoretical theorem that has not resulted from an explanatory inference of any kind. Rather, the number theoretic result belongs to mathematics (of course) and to the *broad* background of this piece of scientific reasoning. The realist can simply *agree* that the description of the scientific inference to the best explanation can in this case make an ineliminable reference to mathematics, *without* altering her justificatory argument which only capitalises on the *narrow* background of the earlier scientific inferences.

To put the point more generally, even if theory choice in science is sometimes made on purely mathematical grounds, and even if mathematics is explanatory in some sense, this utility doesn't have to carry through to the realist commitments because the realist doesn't have to take the *whole* abductive practice of science at face value, but only reap the inferential results of this practice that concern the concrete world. The question of whether mathematics is genuinely explanatory depends on various issues regarding explanation, issues which are certainly not yet completely understood.[14] I don't see any reason to hold the marriage of mathematical nominalism and scientific realism hostage to this question.

7 Conclusion

There are many ways in which mathematics can be indispensable to science. A currently popular way of framing the indispensability argument stresses the role of mathematics in the inductive method of science. This argument is aimed at those sympathetic to scientific realism, for allegedly mathematical entities are on a par with unobservable physical posits of science: both contribute to theoretical desiderata in a way that commands ontological commitment via inference to the best explanation.

This way of motivating platonism as a corollary of scientific realism fails. In the light of the currently popular arguments for scientific realism, there is no tension in believing in the existence of electrons and quarks but not of complex numbers. Although both can play a role in the method of science, the justificatory use of inference to the best explanation particular to scientific realism sets elementary particles apart from numbers. An argument for quarks *qua* an output of reliable scientific reasoning can depend on the role of unobservable concreta in that reasoning, without either similarly depending on the role of mathematics in that reasoning, or denying the role of mathematics altogether.

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Notes

[1]There may be many ways of construing the indispensability argument. Perhaps it can be viewed as a package which includes all sorts of considerations that are not even mentioned here. The present discussion focuses only on this particular way of framing the argument as explicitly directed to the contemporary scientific realist. This version of the argument is very prominent in the literature cited.

[2]As Colyvan puts it: "How are we to understand the phrase 'indispensable to our best scientific theory'? Much hangs on this question, and I'll need to treat it in some detail. ... In fact, whatever

sense it is in which electrons, neutron stars, and viruses are indispensable to their respective theories will do." (2001a, p. 12)

[3]These include, for example, simplicity, unificatory and explanatory power, and fecundity. See Colyvan (2001a, pp. 78–9).

[4]For the purposes of this paper we can understand 'scientific realism' as optimism regarding the arguments rebutting such anti-realist challenges, rather than as optimism about the approximate truth of our best theories, featuring mathematics and all. This statement of realism is more prudent than the familiar motto "our best theories are approximately true," and it serves us better in the present context by more faithfully representing the contemporary realism debate.

[5]In so far as such *qualified* scepticism about the justification of theoretical beliefs about the unobservable is taken seriously, the realist is under pressure to go beyond Quinean scientific realism. The exact character of Quine's realism is difficult to pin down, and I cannot attempt to capture the position in the space available here. What matters for the issue at hand, however, are the idiosyncratic elements of Quine's philosophy, with respect to the general status of knowledge and his empiricism, that are *not* part and parcel of the contemporary realism debate. Although the present argument is not a wholesale repudiation of Quine's brand of holism and naturalism, I think it is safe to say that responding to my argument by appealing to idiosyncrasies of Quinean realism would transform the essence of contemporary scientific realism beyond recognition. For example, as is well known, Quine's ultimate response to underdetermination was to adopt a form of ontological relativism.

[6]Colyvan's claim that scientific realism *tout court* is incompatible with nominalism—that an insistent nominalist can be "forced to scientific *instrumentalism*"—is quite obviously overstated. Due to the grand difficulties in coming up with a credible global justificatory argument for scientific instances of IBE, many have opted for less. It is not clear why the proponents of the various *entity realist* arguments, say, would be at all moved by the kind of descriptive unity that the indispensability argument capitalises on. Ditto for a realist like Kitcher (2001), whose *Galilean strategy* provides another piecemeal justificatory device, turning on an assumption about the uniformity of the success-producing conditions for theorising. But bypassing all these positions can perhaps be considered as a purely rhetorical move; after all, none of them depends on a wholesale espousal of IBE. (Note, though, that advocating a descriptive IBE-model of the scientific method *is* compatible with each of these realist positions.)

Unqualified talk of the realist advocacy of IBE is also unwise. Some realists have put forward arguments that are not meant to apply across the board but only to suitably analysed scientific inferences of a particular form. Lipton (2004) and McMullin (1984), for example, argue that some scientific inferences—via causal and causal-structural explanations, respectively—to the unobservable are of the *same form* as the kinds of inferences to the observable that the anti-realist is happy with. Such restricted 'bridge arguments' for realism, even though they rely on the assumption that scientific reasoning is fundamentally abductive, do not seem to lend themselves to the platonist analogy.

[7]Although Psillos in the above quote speaks of approximate truth of background theories without any anti-platonist qualifications, his more detailed discussion makes it clear that he means truth about concrete theoretical posits. (1999, p. 79)

[8]Something like this was suggested by a referee.

[9]This point also functions as a partial response to Leng (2005) who argues that scientific realism, as formulated by Psillos, for example, is committed to the existence of mathematical abstracta purportedly referred to (or "posited") in our best theories (p. 74). Leng maintains that for this reason, and since the No-Miracles argument by itself fails as an argument for platonism, scientific realists should be *particularly* worried about the platonism/anti-platonism issue. Although I admit that there is room for increased precision in typical statements of scientific realism, in my view there is a natural, more charitable, reading of the realist commitments spoken for by the No-Miracles argument. Scientific realist talk of the "entities posited by a theory", in particular, does not amount to mere quantification over these entities in some preferred logical formalisation of the theory, but takes into account the way these entities are introduced, by scientists, into our belief corpus.

[10]See also Melia (2002), Balaguer (1998, p. 137).

[11]Melia (2000) provides more detailed discussion of the way that mathematics can simplify theories, not the world. These details do not matter to us.

[12]Colyvan (2001a) discusses this point and also provides another response in a Quinean holistic spirit. The latter emphasises the difference in the stances that the nominalist and the platonist hold with respect to gauging the super-empirical virtues of a theory. The nominalist begins with the presupposition that the world *just is* the physical world, and hence the relevant theoretical utility (simplicity, or unification, say), assessed relative to the world, is measured relative to the physical content of the theory. The holist-platonist, on the other hand, looks at the theory more open-mindedly, as it were, and decides upon questions of unification, and explanatory power, say, without such implicit physicalist bias. In effect, the nominalist is accused of question begging.

[13]I find Baker's reasoning seriously flawed at various points, but for the sake of the present argument we can take it showing that there is a sense in which mathematics can be considered genuinely explanatory.

[14]Some accounts of scientific explanation are more amicable to mathematics being explanatory than others. See, for example, De Regt and Dieks (2005).